Spacetime and Gravity: Assignment 7

1. The Schwarzchild metric in coordinates (t, r, θ, ϕ) is given by:

$$ds^2 = -(1 - \frac{2Gm}{r})dt^2 + (1 - \frac{2Gm}{r})^{-1}dr^2 + r^2 d\Omega^2_{(2)} \,. \quad (1)$$

Show that the metric in Eddington-Finkelstein Coordinates $(\bar{t}, r, \theta, \phi)$ with $\bar{t}$ defined as

$$\bar{t} = t + 2Gm\ln(r - 2Gm)\,, \quad (2)$$

is:

$$ds^2 = -(1 - \frac{2Gm}{r})d\bar{t}^2 + (1 + \frac{2Gm}{r})dr^2 + \frac{4Gm}{r}d\bar{t}dr + r^2 d\Omega^2_{(2)} \,. \quad (3)$$

Crucially this metric does not have a singularity at the horizon ie. the metric does not become infinite at the horizon. This demonstrates that the singularity at the horizon of the Schwarchild metric is only a coordinate artifact.

2. The FRW equations for the scale factor R(t) governing a flat expanding universe are given by:

$$\frac{3\dot{R}^2}{R^2} = 8\pi\rho\,, \qquad 2\frac{(\ddot{R}R + \dot{R}^2)}{R^2} = -8\pi p\,. \quad (4)$$

Suppose, the universe was found in some epoch to be expanding such that:

$$R(t) = R_0 t^{\frac{1}{6}}\,. \quad (5)$$

What is the effective equation of state of the matter in the universe during that epoch?

An equation of state is: $p = w\rho$, with w a constant, (it is $\frac{1}{3}$ for radiation and zero for dust).

Hint, solve the FRW equations for a general equation of state where w is an arbitrary constant (not equal to zero) and then find the w required to give the expansion given in equation (5).