Spacetime and Gravity: Assignment 6

1. Consider a spherical shell of mass, M and radius, R.

What will be the metric for $r < R$ ie. inside and for $r > R$ ie. outside the shell?

I wish to crack a code that is encrypted using a prime number factorisation method. Most big banks use these beacuse they are considered completely secure and would take a PC around 100 years to crack by which time the code will change.

Suppose I have access to such a massive shell described above. I place my laptop very far from the shell and set it running to crack the code while I then go and sit inside the shell for a single day thinking about ways to improve the spacetime and gravity course. I then emerge from the shell and check that my laptop has indeed done 100 years of calculations and cracked the Bank's code.

Assuming that in order for me to be comfortable the shell is 10m radius. What is the mass of the shell?

What are the problems with this cunning plan of relieving myself from financial burdens using general relativity?

2. A neutron star has a mass equal to the sun but a radius of just 10km.

Light is emitted by a 300nm source on the surface of the neutron star. What is the wavelength as seen by an observer very far away?

What is the *effective speed of light* of a radially emitted photon at the surface of the star?

3. de Sitter space may be written as a surface embedded in ordinary flat Minkowski space,

$$ds^2 = -dx_0^2 + \Sigma_{i=1}^4 dx^i dx^i \, . \tag{1}$$

The suface obeying the constraint:

$$-x_0^2 + \Sigma_i x^i x^i = 1 \tag{2}$$

defines the de Sitter space.

Consider the coordinates:

$$x_0 = sinh(t) \qquad x_i = cosh(t) y_i \tag{3}$$

What is the constraint in these coordinates? Use this to write down the de Sitter metric. (You may use the notation $d\Omega^2$ to indicate the line element of a sphere.)