Spacetime and Gravity: Assignment 4

In what follows, unless otherwise stated, we will use units such that the speed of light, c=1.

1

The line element of a two dimensional hyperbolic space is given by:

$$ds^2 = \frac{1}{y^2}(dx^2 + dy^2) \tag{1}$$

What is the metric, $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$.

Calculate all the Christofell Symbols for this space? You may use:

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\tau} (\partial_\beta g_{\tau\gamma} + \partial_\gamma g_{\tau\beta} - \partial_\tau g_{\beta\gamma}) \, . \tag{2}$$

Write our the geodesic equations for the hyperbolic space described above.

Calculate the Riemannian curvature of this space. Recall, that only $R_{xyxy} = -R_{yxxy} = -R_{xyyx} = R_{yxyx}$ is non zero.

Calculate the Ricci tensor of this space and show that it solves the vacuum Einstein equation with cosmological constant equal to one. That is:

$$R_{\mu\nu} = -g_{\mu\nu} \, . \tag{3}$$

You can use:

$$R^{\epsilon}{}_{\mu\nu\sigma} = -\partial_\sigma \Gamma^{\epsilon}{}_{\mu\nu} + \partial_\nu \Gamma^{\epsilon}{}_{\mu\sigma} + \Gamma^{\alpha}{}_{\mu\sigma} \Gamma^{\epsilon}{}_{\alpha\nu} - \Gamma^{\alpha}{}_{\mu\nu} \Gamma^{\epsilon}{}_{\alpha\sigma} \tag{4}$$

and

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu} \tag{5}$$