MTH6132, Relativity

Problem Set 7

Due 28th November 2018

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Levi-Civita connection and Riemann curvature

1. Consider the two-dimensional space given in local coordinates $(x^1, x^2) = (x, y)$ by

$$ds^2 = e^y dx^2 + e^x dy^2.$$

(a) Calculate the connection coeffcients.
(b) Calculate the ${R^1}_{212}$ component of the Riemann tensor. Compute also R_{1212}.

2. The metric for a particular 2-dimensional spacetime is given by

$$ds^2 = -e^{2Ar}dt^2 + dr^2$$

where A is an arbitrary constant. Here, we are using the identification $(x^0, x^1) = (t, r)$. From the solution to exercise 4 in problem set 6 we have the non-vanishing Christofell symbols

$${\Gamma^0}_{01} = {\Gamma^0}_{10} = A, \quad {\Gamma^1}_{00} = Ae^{2Ar}.$$

Use this information to calculate the R_{00} component of the Ricci tensor.

3. The Riemann curvature tensor of a certain manifold is of the form

$$R_{abcd} = K\left(g_{ac}g_{bd} - g_{ad}g_{cb}\right),$$

with K a constant. Show that:

(a) Observing that $\nabla_a g_{bc} = 0$, show that $\nabla_e R_{abcd} = 0$. What sort of surface could have a constant curvature?

(b) Show that the corresponding Ricci tensor is proportional to the metric, and that the Ricci scalar is a constant.

4. Suppose that the curvature of a spacetime satisfies the equation

$$R_{ab} - \frac{1}{2}Rg_{ab} + \lambda g_{ab} = 0,$$

where λ is a constant. Show that the Ricci scalar satisfies $R = 4\lambda$. Using this result show then that $R_{ab} = \lambda g_{ab}$.