

MTH6132, RELATIVITY

Problem Set 1

Due 10th October 2018

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The Barn and Pole Paradox

Let a barn be at rest in frame $F : (t, x, y, z)$ have length L in the x direction. Let a pole be at rest in frame $F' : (t', x', y', z')$ that is moving with boost parameter β with respect to frame F . If the pole were at rest in the frame F along with the barn, then it would have a length greater than that of the barn. However, the pole is moving with respect to frame F with precisely the boost parameter β that makes it appear to have exactly the same length L as the barn.

a) In frame F' , what length does the barn appear to have in terms of L and β ? How does this length compare to the proper length L of the barn?

b) In frame F' , what is the proper length of the pole in terms of L and β ? How does the length L of the pole measured in the F frame compare to this proper length?

c) A runner holding the pole in the x direction runs through the barn, whose front and rear doors are open, with a boost parameter β with respect to the F frame. Draw this situation schematically in a spacetime diagram.

d) The barn and pole paradox can be stated as follows. In the F frame the ladder is length contracted to have a measured length L that is precisely the length of the barn. In the F' frame however, the pole is measured to have its proper length which is $> L$, and furthermore the barn is length contracted to have a measured length that is $< L$. In the F frame the situation can be stated as: “the pole fits in the barn”. Construct an analogous statement in the F' frame.

The Twin Paradox

Let the Sol and Alpha Centauri systems both be at rest in frame $F : (t, x, y, z)$. Let the Alpha Centauri system be Δx away from the Sol system as measured in frame $F : (t, x, y, z)$. Let a ship be at rest in frame $F' : (t', x', y', z')$ that is moving with boost parameter β with respect to frame F .

a) What is the distance traveled $\Delta x'$ as measured in the F' frame in terms of Δx and β ?

b) What is the elapsed time of travel $\Delta t'$ as measured in the F' frame in terms of Δx and β ?

c) What is the elapsed time of travel Δt_0 as measured in the F frame in terms of Δx and β ?

d) The ship travels from the Sol system to the Alpha Centauri system at constant boost β , and as soon as it arrives it immediately travels from the Alpha Centauri system to the Sol system at the same constant boost β . The two trajectories of the twins have the same endpoints: label these endpoints p and q . Draw this situation schematically in a spacetime diagram.

e) The twin paradox can be stated as follows. Twin A stays in the Sol system while twin B travels to the Alpha Centauri system with the trajectory described in (d). Twin A expects that the other has experienced more elapsed time, while twin B also expects that the other has experienced more elapsed time. State which twin is correct, and explain why by comparing the spacetime interval between the same two endpoints p and q but measured along the two different trajectories of twin A and twin B.