Spacetime and Gravity: Assignment 6 Solutions

November 8, 2013

1.

The metric outside the shell will be Schwarzchild,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})dt^2 + (1 - \frac{2GM}{c^2 r})^{-1} dr^2 + r^2 d\Omega_2^2 \quad (1)$$

The metric inside must be that of flat space, since there is no matter inside (think of Gauss's law) BUT it must smoothly join onto the spacetime outside of the shell at $r = R$. This means the metric at the shell ie. $r = R$ outside must equal the metric for a flat spacetime inside. This means the metric inside the shell is:

$$ds^2 = -(1 - \frac{2GM}{c^2 R})dt^2 + (1 - \frac{2GM}{c^2 R})^{-1} dr^2 + r^2 d\Omega_2^2 \quad (2)$$

where in this case R=10m.

Now, I will move in proper time, s inside the shell, but the outside world will move in time t_∞ and so since one day inside is one hundred years outside:

$$ds = \sqrt{(1 - \frac{2GM}{c^2 R})} dt_\infty = (100 \times 365)^{-1} dt_\infty \quad (3)$$

And so,

$$\frac{2GM}{c^2 R} = 1 - (100 \times 365)^{-2} \quad (4)$$

then,

$$M = \frac{(c^2 R)}{2G}(1 - (100 \times 365)^{-2}) \quad (5)$$

finally putting $R = 10m$, $c = 3 \times 10^8$, $G = 7 \times 10^{-11}$ gives:
$M = 6 \times 10^{27} kg$.

2.

Follows the course notes done in class:

$$\frac{\lambda_r}{\lambda_\infty} = \sqrt{(1 - \frac{2GM}{c^2 r})} \quad (6)$$

Put in the mass of the sun $M = 2 \times 10^{30} kg$ and other constants:

$$\frac{\lambda_r}{\lambda_\infty} = 0.83 \tag{7}$$

So the wavelength far away is $\lambda_{infty} = 300/0.83 = 361nm$.

The so called *effective speed of light* will be given by the condition of $ds^2 = 0$ because light null and $d\theta = 0$, $d\phi = 0$ for a radial ray:

$$\frac{dr}{dt} = 1 - \frac{2GM}{c^2 R} \tag{8}$$

And then evaluate for $R = 10000m$ and $M = 2 \times 10^{30} kg$.

3.

Insert the coordinates given in equation (3) of the question into the constraint given by equation (2):

$$-sinh^2(t) + cosh^2(t) \sum_i y_i y^i = 1 \tag{9}$$

which in tern implies

$$\sum_i y_i y^i = 1\,. \tag{10}$$

This is the equation for a unit sphere. And so instead of writting out the surface embedded in flat Minkowski we solve the constraint to give the metric:

$$dx_0^2 = -cosh^2(t) dt^2 \tag{11}$$

$$dx_i = sinh(t) dt y_i + cosh(t) dy_i \tag{12}$$

$$\begin{aligned} dx_i dx^i &= (sinh(t)dty_i + cosh(t)dy_i)(sinh(t)dty^i + cosh(t)dy^i) \quad (13) \\ &= y_i y^i sinh^2(t) dt^2 + cosh^2(t) dy_i dy^i + 2sinh(t)cosh(t) dy_i y^i \quad (14) \end{aligned}$$

The use $y_i y^i = 1$ differentiate this implies $y_i dy^i = 0$ and insert into the above and finally use $cosh^2(t) - sinh^2(t) = 1$. to give:

$$ds^2 = -dt^2 + cosh^2(t) d\Omega^2 \tag{15}$$

where $d\Omega$ is the line element of a sphere.