Spacetime and Gravity: Assignment Solutions 5

November 11, 2013

1.

First workout the metric in these coordinates:

$$g'_{tt} = g_{tt}\frac{\partial t}{\partial t'}\frac{\partial t}{\partial t'} + g_{xx}\frac{\partial x}{\partial t'}\frac{\partial x}{\partial t'} = -1 + (v + at')^2 \quad (1)$$

$$g'_{xt} = g_{xx}\frac{\partial x}{\partial x'}\frac{\partial t}{\partial t'} = v + at' \quad (2)$$

$$g'_{xx} = g_{xx}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'} = 1 \quad (3)$$

Then the inverse metric:

$$g'^{tt} = -1 \quad (4)$$

$$g'^{xt} = (v + at') \quad (5)$$

$$g'^{xx} = 1 - (v + at')^2 \quad (6)$$

Now work out the x, t, t, Christofell symbol:

$$\Gamma^x{}_{tt} = \frac{1}{2}g'^{xx}(2\partial_t g'_{tx}) + \frac{1}{2}g'^{tx}\partial_t g'_{tt}) \quad (7)$$

$$= \frac{1}{2}(1 - (v + at')^2)2a + \frac{1}{2}(v + at')^2 2a \quad (8)$$

and so

$$\Gamma^x{}_{tt} = a \quad (9)$$

In the nonrelativistic we take $s = t$, So the geodesic equation for x in the nonrelativistic limit is:

$$\frac{d^2x}{dt^2} = -\Gamma^x{}_{tt} = -a \quad (10)$$

2.

The rotating coordinates coordinate system will be related to static coordinates as follows:

$$\theta' = \omega t + \theta \qquad t' = t \qquad r' = r \quad (11)$$

Then the metric in these coordinates is:

$$\begin{aligned} g'_{tt} &= -1 + r^2\omega^2 \quad (12)\\ g'_{t\theta} &= -\omega r^2 \quad (13)\\ g'_{\theta\theta} &= r^2 \quad (14)\\ g'_{rr} &= 1 \quad (15)\\ g'_{rt} &= g'_{r\theta} = 0 \quad (16)\\ & \quad (17) \end{aligned}$$

Now the Christofell for r, t, t components:

$$\Gamma^r{}_{tt} = -\frac{1}{2} g^{rr} \partial_r g'_{tt} = -r\omega^2 \quad (18)$$

In the nonrelativistic limit where $s = t$ the geodesic equation for r is:

$$\frac{d^2 r}{dt^2} = -\Gamma^r{}_{tt} = r\omega^2 \quad (19)$$

This is exactly the centrepedal force!