PHY308: Space, Time, and Gravity - Week 3 Homework

In what follows, unless otherwise stated, we will use units such that the speed of light, c=1.

Problem 1 (25 marks)

Take the following 2 tensor:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \tag{1}$$

and the following vector:

$$j^{\mu} = \begin{pmatrix} Q \\ j_x \\ j_y \\ j_z \end{pmatrix} \tag{2}$$

Now calculate:

$$\partial_{\mu} F^{\mu\nu} = j^{\nu} \tag{3}$$

in terms of the vectors: $\mathbf{E} = (E_x, E_y, E_z)$, $\mathbf{B} = (B_x, B_y, B_z)$, $\mathbf{j} = (j_x, j_y, j_z)$ and the scalar Q.

Do you recognise these equations? (The answer yes or no will not do).

(Hint, write out the components of $curl\mathbf{B}$.)

Problem 2 (25 marks)

The line element on a unit 2-sphere is:

$$ds^2 = d\theta^2 + sin(\theta)^2 d\phi^2 \tag{4}$$

What is the metric? [2 marks]

What is the inverse metric? [3 marks]

What are the Christoffel symbols? [20 marks]

You will need

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\delta} (\partial_{\gamma} g_{\delta\beta} + \partial_{\beta} g_{\delta\gamma} - \partial_{\delta} g_{\beta\gamma}) \tag{5}$$