Quantum Physics PHY4215 - Exercise Sheet 8

1. A linear operator $\hat{A}$ is a transformation of wavefunctions which has the following properties

$$\hat{A}(c_1\psi_1 + c_2\psi_2) = c_1(\hat{A}\psi_1) + c_2(\hat{A}\psi_2)$$

Verify that the operators corresponding to position, momentum, energy in quantum mechanics are linear.

Explain how these along with the non-relativistic formula $E = \frac{p_x^2}{2m} + U(x)$ are used to derive the time-dependent Schrodinger equation for the particle.

2. A particle moves in 2 dimensions, under the influence of a potential energy function $U(x,y) = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2y^2$. Derive the time-dependent Schrodinger equation for this particle.

3. The postulates of quantum mechanics say that a physical observable A corresponds to an operator $\hat{A}$. If $\psi_a(x)$ is an eigenstate of $\hat{A}$, with eigenvalue a :

$$\hat{A}\psi_a = a\psi_a$$

then the measurement of A gives, with certainty, the value a.

(a) Consider the wavefunction $e^{ikx-i\omega t}$ for constants k, ω. By considering the eigenvalue postulate above, explain how we can recover the de Broglie equations for energy and momentum of a particle in terms of the wave properties of frequency and wavelength. **[6]**

(b) Consider wavefunction $e^{ik_xx+ik_yy-\frac{y^2}{2d^2}+i\omega t}$, for constants k_x, k_y, ω. Does a system in this quantum state have a definite momentum in the x, y, z directions ? If so, what are they ? Does it have a definite energy ? **[8]**