Quantum Physics PHY4215 - Exercise Sheet 6

1. The Gaussian wavepacket is a wave with variable amplitude

$$\Psi(x) \equiv \Psi_{k_0,d}(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{d}} e^{ik_0 x - \frac{x^2}{2d^2}}$$

Show that this is a normalized wavefunction, i.e.

$$\int_{-\infty}^{\infty} \Psi(x)\Psi^*(x)dx = 1$$

You may use the integral

$$\int_{-\infty}^{\infty} dx \;\; e^{-ax^2} = \sqrt{\frac{\pi}{a}}$$

2. Consider the integral

$$< x >= \int_{-\infty}^{\infty} x\Psi(x)\Psi^*(x)dx$$

Use the change of variable $x' = -x$ to show that $< x >= - < x >$, hence that $< x >= 0$.

3. A probability density $P(x)$ is positive function which is normalized to one, $\int_{-\infty}^{\infty} P(x)dx = 1$. The expectation value $< f(x) >$ of any function $f(x)$ is defined as

$$\int_{-\infty}^{\infty} f(x)P(x)dx =< f(x) >$$

Show that

$$< \;\; (x- < x >)^2 \;\; >=< x^2 > - < x >^2$$

which says that the mean squared deviation is equal to the difference between the mean of the square and the square of the mean. The quantity $(\Delta x)^2 =< \;\; (x- < x >)^2 \;\; >$ is a measure of the spread of x away from the mean when the measurements of x are distributed according to the probability distribution $P(x)$.

4. Show that, for the probability distribution $P(x) = \Psi^*(x)\Psi(x)$,

$$(\Delta x) = \frac{d}{\sqrt{2}}$$

Remark : You may wish to use the differentiation trick from class to calculate $< x^2 >$.