Quantum Physics PHY4215 - Exercise Sheet 2

1. Using Planck's formula for the blackbody spectral distribution,

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

we can integrate over wavelengths to get the Stefan-Boltzmann Law,

$$P = \sigma T^4$$

with Stefan's constant

$$\sigma = 5.67 \times 10^{-8} W \mathrm{m}^{-2} K^{-4}$$

This derivation gives

$$\sigma = \frac{2\pi^5}{15} \frac{k^4}{h^3 c^2}$$

By using the peak of $R(\lambda, T)$, the Wien's displacement constant b (known to be $b = 2.90 \times 10^{-3}\mathrm{mK}$) which is related to λ_{max} via the equation

$$\lambda_{max} T = b$$

is found to be

$$b = \frac{hc}{4.965k}$$

Use the equations for b, σ in terms of h, c, k, to express h, k in terms of b, σ, c. Using the values of b, σ and $c = 3.00 \times 10^8 \mathrm{ms}^{-1}$, calculate the numerical Values of h, k.

2. In the lectures, we worked with the spectral emittance given in terms of the wavelength $R(\lambda, T)$. We could equally work with a spectral emiittance $\tilde{R}(f, T)$ defined as a function of frequency f, where $c = f\lambda$ and c is the speed of light. These two functions are related by the requirement that an interval of size $d\lambda$ in wavelengths is related to an interval of size df in frequencies, by the equations

$$df = \frac{df}{d\lambda} d\lambda$$

and the power emitted (per unit area, per unit wavelength range) is the same whether described in terms of wavength range or corresponding frequency range,

$$\tilde{R}(f, T)\ |df| = R(\lambda, T)\ |d\lambda|$$

The quantities $|df|$ and $|d\lambda|$ are the magnitudes of small intervals in df in frequency and $d\lambda$ in wavelength. Use this to show that

$$\tilde{R}(f, T) = \frac{2\pi hf^3}{c^2} \frac{1}{e^{\frac{hf}{kT}} - 1}$$

[10]

Remark : Note that $\tilde{R}(f, T)$ is not equal to $R(\lambda, T)$ with λ set to $\frac{c}{f}$.